Title: Appraising vibro-settlement prediction methods using the finite element method

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Abstract

Numerous approaches exist for the prediction of the settlement improvement offered by the vibro-replacement technique in weak or marginal soil deposits. The majority of the settlement prediction methods are based on the unit cell assumption, with a small number based on plane strain or homogenization techniques. In this paper, a comprehensive review and assessment of the more popular settlement prediction methods is carried out with a view to establishing which method(s) are in best agreement with finite element predictions from a series of PLAXIS 2D axisymmetric analyses on an end-bearing column. The Hardening Soil Model in PLAXIS 2D has been used to model the behaviour of both the granular column material and the treated soft clay soil. This study has shown that purely elastic settlement prediction methods overestimate the settlement improvement for large modular ratios while the methods based on elastic-plastic theory are in better agreement with finite element predictions at higher modular ratios (in some cases owing to the assumption of a significant bulging mechanism which is more prevalent in soft soils and in other cases due to the variability of soil stiffness with stress level which is considered in the FE calculations but not in the analytical solutions). In addition, a parameter sensitivity study has been carried out to establish the influence of a range of different design parameters on predictions obtained using a selection of elastic-plastic methods.

1. Introduction

The vibro-replacement (stone column) technique has become increasingly popular in recent years as a method of treating weak or 'marginal' soil deposits. The potential of the technique to reduce settlement [1], improve bearing capacity [2], accelerate consolidation [3] and reduce the likelihood of liquefaction [4] is now widely accepted in geotechnical practice. The vibro-replacement technique and associated equipment have been described in detail by [5] and [6]. McCabe *et al.* [7] have used a database of field settlements to illustrate that the 'bottom-feed' system (in which stone is added through a delivery tube along the side of a vibrating poker and exits at the poker tip) produces consistently higher settlement improvement factors than other systems in soft or fine soils.

Most analytical design methods involve the direct prediction of a settlement improvement factor, n, defined as the settlement of untreated ground divided by the settlement of the ground treated with granular columns ($n = s_0/s_t$), before subsequently applying this improvement factor to predict the settlement of treated ground ($s_t = s_0/n$). The value of s_0 (for wide-area loading) is usually calculated from elastic theory as $s_0 = p_aH/E_{oed}$, where p_a is the applied pressure, H is the thickness of the treated soil layer, and E_{oed} is the oedometric soil modulus. The analytical design methods typically relate n to the area-replacement ratio, A_c/A (where A is the cross-sectional area of a unit cell treated with a single stone column of cross-sectional area, A_c, see Figure 1). The area-replacement ratio is a measure of the amount of in-situ soil replaced with stone and is dependent on the column spacing, s and column diameter, D (Equation 1), where k is a constant depending on the column arrangement (Figure 1).

$$\frac{A}{A_c} = k\left(\frac{s}{D}\right)^2 \tag{1}$$

A number of other influential variables have been identified by analytical studies, and these include effect of installation, load level, modular ratio and the friction and dilatancy angles of the column material. The published solutions account for these variables in different ways, although few capture all of them. The aim of this study is to provide a systematic review of these methods before using a 2D/axisymmetric finite element (FE) parametric study to appraise the ability of different methods to cater for the variables identified above. The Hardening Soil (HS) Model is used in conjunction with PLAXIS 2D (Brinkgreve *et al.* [8]) to model a soft soil profile and field settlement improvement factors published by McCabe *et al.* [7] provide some context for both the analytical and numerical analyses.

2. Vibro-Replacement Settlement Prediction Methods

2.1 Theoretical Considerations in Vibro-Replacement Design

Settlement design approaches tend to be either elastic (e.g. [9-12]), where yielding of the column material is not considered, or elastic-plastic (e.g. [13-21]). The elastic-plastic methods are based on the Mohr Coulomb failure criterion, with some assuming that the granular material deforms at constant volume as it yields (dilatancy angle, $\psi = 0^{o}$), while others have accounted for dilation of the granular column material at yield using a constant dilatancy angle. Balaam and Booker [15] and Pulko and Majes [18] have highlighted that elastic-plastic methods are preferable to purely elastic methods because the elastic methods tend to over-predict the settlement improvement offered by column installation, especially for high modular ratios (E_c/E_s, where E_c is the modulus of the column and E_s is the modulus of the soil). This over-prediction is as a result of the fact that elastic methods over-predict the stress concentration factor (SCF = σ_c/σ_s, where σ_c is the stress in the column and σ_s is the stress in the soil).

Approaches to modelling the behaviour of the column-soil system vary; some, such as Han and Ye [12] have accounted only for vertical deformation, while others have accounted for both radial and vertical deformation. For elastic methods that consider vertical deformation only, the SCF is equal to the ratio of the oedometric moduli. Elastic solutions that consider both radial and vertical deformation result in slightly lower SCFs (lateral deformation reduces SCFs, e.g. Castro and Sagaseta [20]). However, these SCFs will still be too high because yielding of the column material is not considered (column yielding and plastic strains will reduce SCFs). Barksdale and Bachus [2] have suggested that commonly encountered SCFs in practice range from 3-10 depending on the column spacing adopted in the field.

The Priebe [13] and Goughnour and Bayuk [14] solutions are formulated on the assumption that the granular column material is incompressible. Most neglect immediate settlement; Baumann and Bauer [9] and Castro and Sagaseta [20] are notable exceptions. The densification effect resulting from column installation and subsequent bulging has been accounted for in different ways. Priebe [13] has assumed an increase in the coefficient of lateral earth pressure following column installation to the liquid earth pressure of the soil (K = 1). Other methods allow for the input of different values depending on the designer's discretion: Baumann and Bauer [9] have limited allowable K values to the range $K_0 < K < 1/K_0$; Goughnour and Bayuk [14] have limited allowable K values to the range $K_0 < K < K_p$, where K_0 and K_p are the at-rest and passive earth pressure coefficients of the soil respectively; Borges *et al.* [19] have formulated their closed-form expression based on best-fitting curves to the results of numerical analyses assuming $K = 0.7$ (between the conservative, $K = 1 - \sin \varphi'$ for normally consolidated soils, and $K = 1$ approaches); Van Impe and Madhav [16] have suggested the use of an increased oedometric soil modulus depending on the method of installation and the column spacing.

Solutions have been developed for drained conditions and for undrained conditions with a follow-up consolidation period to allow for the dissipation of excess pore pressure. The undrained plus consolidation solutions (e.g. Han and Ye [12], Castro and Sagaseta [20]) have been based on Barron's [22] solution for vertical drains (Barron's [22] solution assumes that the vertical stress on the soil is constant during the consolidation process), but with modified coefficients of consolidation used to account for the fact that the columns carry a considerable proportion of the applied load (vertical drains have a much smaller stiffness and diameter than stone columns). Castro and Sagaseta's [20] solution has been derived for the case of an elastic-plastic column (radial deformation has been considered) while Han and Ye [12] have based their solution on an elastic column subjected to full lateral confinement (i.e. no radial strain). A selection of settlement design methods and their inherent assumptions have been summarised in Table 1.

2.2 General Settlement Prediction Approaches

Greenwood [23] was the first to present a means of estimating the settlement improvement achievable using the vibro-replacement technique. Based on the column spacing, the construction technique (i.e. wet/dry method), and the undrained shear strength of the treated soil, Greenwood [23] presented a set of empirical curves for the estimation of the extent of settlement improvement, noting that precise mathematical solutions had not yet been developed at the time. Similar to the analytical solutions that have been developed in the interim, Greenwood's [23] curves have been proposed for end-bearing columns neglecting immediate settlements and shear displacements (considering that Greenwood's curves are an empirical proposal, it is not precisely clear to the authors why these factors have been neglected).

At present, the majority of the design methods have been derived for a unit cell representing an infinite grid of regularly spaced end-bearing columns, e.g. [9-21]. The unit cell approach is based on the assumption of a large grid of regularly spaced columns subjected to a uniform load. Therefore all of the columns will exhibit similar behaviour and an analysis of one such column and its tributary soil area is sufficient. Owing to the symmetry of the problem, the shear stresses along the perimeter of the unit cell are assumed to be zero. The unit cell approach is valid except for columns near the edges of the loaded area [11,24], which are assumed to be in the minority for large groups.

Other solutions have also been developed based on plane strain (e.g. Van Impe and De Beer [25]) or homogenization techniques (e.g. Schweiger and Pande [26], Lee and Pande [27]). The plane strain approach involves replacing the stone columns with stone walls (trenches) having an 'equivalent' overall plan area. The homogenization technique involves modelling the stone column and treated soil as a composite material with improved soil properties and is formulated assuming that the influence of the columns is uniformly and homogeneously distributed throughout the treated soil, e.g. [26].

For all three approaches, further simplifying assumptions are usually considered, e.g. the column and the surrounding soil undergo equal vertical settlement, i.e. $\delta_c = \delta_s$, where δ_c and δ_s are the settlements of the columns and soil respectively (referred to as the 'equal vertical strain' assumption) and the shear stresses at the column-soil interface are assumed to be zero. The homogenization technique can be used in conjunction with flexible and rigid rafts ('equal vertical stress' and 'equal vertical strain' assumptions, respectively), which makes it possible to isolate different behavioural aspects associated with columns near the edge of a loaded area. It can also be used to model the behaviour of floating columns (the plane-strain and unit cell approaches are generally based on end-bearing stone columns). However, they can be used to model floating columns in conjunction with FE analyses (FE solutions generally assume that there is no slip at the column-soil interface).

2.3 Unit Cell Approaches

The simplest analytical approach to stone column design is known as the 'equilibrium method'. The approach is based on elastic theory and has been described by Aboshi *et al.* [10]. It is based on vertical equilibrium between the soil and the columns with oedometric (i.e. elastic behaviour with full lateral confinement) conditions in the soil. From vertical equilibrium (Equation 2):

$$\boldsymbol{p_a} \cdot \boldsymbol{A} = \sigma_{\boldsymbol{c}} \cdot \boldsymbol{A_c} + \sigma_{\boldsymbol{s}} \cdot (1 - \boldsymbol{A_c}) \tag{2}$$

The settlement (assuming oedometric conditions) is then calculated as: $s = \sigma_s.H/E_{oed}$. The settlement improvement factor (n) is calculated as s_0/s (and rearranging gives the expression in Equation 3), where $s_0 = p_a.H/E_{oed}$ as defined earlier. This approach necessitates prior knowledge of the SCF (e.g. experience/field measurements) whereas other methods such as Priebe [13,17] have used cylindrical cavity expansion (CCE) theory to establish the SCF. The method by Aboshi *et al.* [10] limits allowable SCFs based on the friction angles of the soil and column materials and the undrained shear strength of the soil.

$$\boldsymbol{n} = 1 + \frac{\boldsymbol{SCF} - 1}{\boldsymbol{A} / \boldsymbol{A_c}} \tag{3}$$

Balaam and Booker [11] have adopted an elastic approach based on a unit cell of effective diameter, d_e, which is dependent on the column spacing (s) and whether the columns are arranged on either triangular (d_e = 1.05s), square (d_e = 1.13s), or hexagonal grids (d_e = 1.29s). Balaam and Booker [15] have extended the 1981 solution using an interaction analysis to account for yield of the granular material. The clay is assumed to behave elastically while the stone is assumed to behave as a perfectly elastic-plastic material (non-associative flow rule) satisfying the Mohr Coulomb failure criterion. Elasto-plastic FE analyses were performed to validate the assumptions inherent in the interaction analysis. Balaam and Booker's [11] method can be used to obtain a closed-form analytical solution while Balaam and Booker's [15] method is an iterative approach requiring numerical implementation to obtain a solution.

Goughnour and Bayuk [14] have formulated an elastic-plastic method based on a unit cell of effective diameter, $d_e = 1.05s$ (triangular grid of columns). The method is alternatively referred to as the 'incremental method' and is an extension of earlier solutions developed by Baumann and Bauer [9], Hughes *et al.* [28] and Priebe [13]. As consolidation proceeds, stresses are gradually transferred from the soil to the column. Two sets of analyses have been performed, considering both elastic and plastic behaviour of the column material. Firstly, an analysis is performed assuming that the stone undergoes plastic deformation while the surrounding soil undergoes consolidation. A second analysis is performed assuming the stone to behave elastically up until the end of consolidation. The vertical strains (ε_v) evaluated using the two methods are compared. The long-term vertical strain is then taken to be the larger of the two values, and the resulting settlement, δ, can be calculated as $\delta = \varepsilon_v.H$, where H is the layer thickness. Baumann and Bauer's [9] analytical elastic approach was developed assuming the total settlement of the loaded soil layer to consist of the immediate settlement (no volume change) and the consolidation settlement.

Despite its heavily empirical basis, Priebe's [17] method has become one of the most popular design methods (European practice) for evaluating the settlement improvement factor associated with vibro-improved ground. Priebe's [17] method is an extension of Priebe's [13] method in which CCE theory has been used to evaluate the radial strain assuming zero vertical strain (and hence the SCF). The vertical strain was first evaluated assuming zero radial strain. The densification of the surrounding soil as a result of column installation has been accounted for by using an increased coefficient of lateral earth pressure ($K = 1$) in the design procedure. Priebe [13] makes a number of simplifying assumptions to calculate a 'basic' improvement factor, n_0, as defined in Equation 4, assuming a Poisson's ratio for the soil, v_s, of 0.33 (the method allows for different Poisson's ratios) for the soil, where φ'_c is the friction angle of the granular material. In the calculation of n_0, it is assumed that bulging is constant over the length of the column, the column material is incompressible, and the bulk densities of the soil and column are neglected.

$$\boldsymbol{n}_0 = 1 + \frac{\boldsymbol{A_c}}{\boldsymbol{A}}\left[\frac{5 - \frac{\boldsymbol{A_c}}{\boldsymbol{A}}}{4.\left(1 - \frac{\boldsymbol{A_c}}{\boldsymbol{A}}\right).\tan^2\left(45 - \frac{\phi'_c}{2}\right)} - 1\right] \tag{4}$$

Priebe's [17] method accounts for the column compressibility (n_1) and the bulk densities of the soil and column materials (n_2). Consideration of the compressibility of the column material means that load application can result in settlement that is unrelated to column bulging. The calculation of n_1 involves 'shifting' (based on the modular ratio) the n_0 curve to work out a value $\Delta(A/A_c)$. $\Delta(A/A_c)$ is then added to A/A_c and a new improvement factor is evaluated. Consideration of the soil and column unit weights (n_2) means that the columns are provided with more lateral support (hence increasing the bearing capacity of the composite system). Neglecting the bulk densities implies that bulging would be constant over the length of the column (because the initial pressure difference between the columns and the soil which leads to bulging will be constant over the length of the column). However, consideration of the soil and column weights means that the initial pressure difference between the columns and soil will decrease asymptotically with depth thus leading to a reduction of bulging with depth. Priebe's [17] n_2 also allows for the input of different K values by modifying the depth factor, f_d, used in the calculation of n_2.

The elastic-plastic methods derived by Pulko and Majes [18], Castro and Sagaseta [20], and Pulko *et al.* [21] account for dilation of the granular column material (constant dilatancy angle, ψ) at yield whereas Priebe's [13,17] method assumes the granular column material to deform at constant volume (ψ = 0°). Pulko and Majes [18] and Castro and Sagaseta [20] are elastic-plastic extensions of the earlier elastic solution developed by Balaam and Booker [11] for drained conditions. Castro and Sagaseta [20] have considered an undrained loading situation followed by a consolidation process to allow for the dissipation of excess pore pressures whereas Pulko and Majes [18] and Pulko *et al.* [21] have studied the unit cell problem under drained conditions. As noted by Castro and Sagaseta [29], both approaches are considered to be limiting cases of the real situation because load application is not rapid enough to be considered as undrained nor slow enough to be considered as a drained process.

The method developed by Pulko *et al.* [21], which deals with encased stone columns, is an extension of the previous solution derived by Pulko and Majes [18]. The new method by Pulko *et al.* [21] can also be applied to non-encased stone columns by setting the encasement stiffness to zero. The solutions derived by Castro and Sagaseta [20] and Pulko and Majes [18] ignored the elastic strains in the column during its plastic deformation whereas the newer solution by Pulko *et al.* [21] has taken them into account.

Figure 2 (from Castro and Sagaseta, [29]) shows the different stress paths followed depending on whether the problem is studied under drained or undrained (plus consolidation) conditions. For the case of an elastic column, both approaches produce the same result. For a yielding column (elastic-plastic case), although the stress paths are different, the 'real' results, e.g. the final settlements, are very similar (providing that the drained solutions account for elastic strains of the column during its plastic deformation), as shown by Castro and Sagaseta [29] using finite element calculations. For drained analyses that neglect the elastic strains of the column during its plastic deformation (e.g. Pulko and Majes [18]), the final settlement will be underpredicted. For undrained plus consolidation solutions (e.g. Castro and Sagaseta [20]), neglecting the elastic strains of the column during its plastic deformation leads to negligible error in the solution. The newer drained solution by Pulko *et al.* [21] accounts for the elastic strains of the column during its plastic deformation. Under such conditions, the differences between the drained and undrained (plus consolidation) analyses will effectively vanish (i.e. Castro and Sagaseta [20] and Pulko *et al.* [21] will produce almost identical solutions for non-encased columns, as studied here).

The design methods derived by Castro and Sagaseta [20] and Pulko *et al.* [21] have dealt with column yielding in different ways. Castro and Sagaseta's [20] undrained plus consolidation formulation uses a factor U_y^e (elastic degree of consolidation at the moment of column yielding) to work out whether or not the column is in a plastic state (if $U_y^e > 1$, no yielding takes place, otherwise yielding of the granular material occurs). Pulko *et al.* [21] have worked out a final yield depth, z_y (i.e. yielding starts at the surface and progresses downward as the applied load increases), to which plastic strains appear in the column.

Borges *et al.* [19] have proposed a design method (based on a numerical rather than an analytical approach) relating the settlement improvement factor (n) to the area-replacement ratio, A_c/A, and to the ratio of the deformability of the soft soil to the deformability of the column material (alternatively the modular ratio, E_c/E_s). Their resulting design equation (and design chart) is based on curve-fitting to the results of a series of axisymmetric FE analyses of a unit cell with a program incorporating Biot consolidation theory with the p-q-θ model (extension of the Modified Cam-Clay (MCC) Model, based on the Drucker-Prager failure criterion). In contrast to the MCC Model, the parameter M (defining the slope of the critical state line) is not constant, e.g. Lewis and Schrefler [30], Domingues *et al.* [31].

The authors have adopted a value of $K = 0.7$ for the coefficient of lateral earth pressure at rest following column installation (in between $K = 1 - \sin \varphi'$ and $K = 1$). The settlement improvement factor (Equation 5) has been derived based on statistical analysis techniques; and has been related to the two factors that the authors found had the most significant influence on the results. A design chart has been developed based on this design equation, which is applicable for $10 \leq E_c/E_s \leq 100$ and $3 \leq A/A_c \leq 10$, with calculated improvement factors greater than 1.5.

$$\boldsymbol{n} = \left(0.125 \frac{\boldsymbol{E_c}}{\boldsymbol{E_s}} + 0.7742\right)\left(\boldsymbol{A} / \boldsymbol{A_c}\right)^{\left(-0.0038 \frac{E_c}{E_s} - 0.3423\right)} \tag{5}$$

A flow chart detailing the development and origin of the majority of the design methods based on the unit cell approach is presented in Figure 3.

3. Axisymmetric Modelling (PLAXIS 2D)

Axisymmetric FE analyses using PLAXIS 2D (Brinkgreve *et al.* [8]) have been carried out as a means of appraising the capabilities of several of the aforementioned analytical methods. A unit cell approach (Figure 4) with a column radius, $R_c = 0.3$ m (typical for columns at soft soil sites, e.g. Watts *et al.* [1]), and a column length = 5 m has been adopted to represent the behaviour of a single end-bearing column within an infinite grid. Similar modelling approaches have been adopted by Debats *et al.* [32] and Ambily and Gandhi [33]. Horizontal deformation has been restricted at the sides (roller boundaries) and both vertical and horizontal deformations have been restricted at the base. The water table is located at the surface. The columns are fully penetrating and have been wished in place (as is common practice), e.g. Gäb *et al.* [34] and Killeen and McCabe [35]. For the initial study, the coefficient of lateral earth pressure, K, is assumed to be unaffected by column installation ($K_0 = 1 - \sin \varphi' = 0.44$). A parameter sensitivity study considering different K values has been described in section 4.3.4, e.g. Priebe [13], Goughnour and Bayuk [36] and Gäb *et al.* [34] have accounted for the densification as a result of column installation by using an increased coefficient of lateral earth pressure, $K = 1$ (for the soil).

The behaviour of the composite model has been studied under a 100 kPa load (the sensitivity study described in section 4.3.1 has also examined the behaviour of the system under 50 kPa and 75 kPa loads) applied through a plate element (normal stiffness, $EA = 5\text{x}10^6$ kN/m, flexural rigidity, $EI = 8.5\text{x}10^3$ kNm^2/m, Poisson's ratio, ν =

0). The plate element is intended to represent a rigid loading platform to prevent differential settlements. Different series of analyses have been carried out for different modular ratios, E_c/E_s, of 5, 10, 20 and 40 (note that good comparison with elastic methods necessitates the use of low E_c/E_s ratios). These values of E_c/E_s are in the same range as those adopted by Balaam and Booker [11], Castro and Sagaseta [20], and Poorooshasb and Meyerhof [37]. In all cases, the properties of the column material have been fixed while the soil properties have been varied to generate the necessary E_c/E_s ratios. The diameter of the unit cell has been altered to study the effect of different area-replacement ratios, e.g. Domingues *et al.* [31]. The column diameter has been fixed at 0.6 m (arguably the column diameter in the field will be a function of E_c/E_s but a fixed diameter has been considered here for numerical purposes).

Load settlement behaviour (primary settlement) has been analysed using the HS Model to model both the clay and the stone. Both have been modelled as fully drained materials. Similar results would be achieved modelling the clay as an undrained material with a follow-up consolidation period (analyses have been carried out in verification, e.g. Figure 5).

The HS Model is a hyperbolic elastoplastic model that accounts for increasing soil layer stiffness with stress level (no viscous effects). Its formulation has been described in detail by Schanz *et al.* [38]. A friction angle (φ') of 45^o has been selected for the stone, representative of bottom feed columns, while the dilatancy angle (ψ) was calculated as $\psi = \varphi' - 30^o$. E_{oed}^{ref} (oedometric modulus) was assumed approximately equal to E_{50}^{ref} (secant modulus) and E_{ur}^{ref} (unload-reload modulus) was taken as $3E_{50}^{ref}$, as recommended by Brinkgreve *et al.* [8]. The values of E_{oed}^{ref}, E_{50}^{ref}, and E_{ur}^{ref} for the stone quoted in Table 2 are based on Gäb *et al.* [34]. The properties have been altered using Equation 6 to correspond to a confining pressure, σ'_3, of 50kPa (closer to the confining pressure in the subsequent numerical simulations). Gäb *et al.* [34] have defined the stiffness moduli at a reference pressure, p^{ref}, of 100 kPa. The stress dependency of soil stiffness is dictated by the power, m (m = 1 is typical for soft soils [8]). For the granular column material, a value of m = 0.3 has been used [34].

$$E = E^{ref}\left(\frac{c'\cos\phi' + \sigma'_3\sin\phi'}{c'\cos\phi' + p^{ref}\sin\phi}\right)^m \tag{6}$$

A complete list of the parameters used in the FE model for the case when E_c/E_s = 20 is given in Table 2. The E_c/E_s ratio has been defined as the ratio of the constrained/oedometric moduli at a reference pressure of 50kPa, i.e. at p^{ref} = 50 kPa, $E_{oed,c}/E_{oed,s}$ = 56,858/2,843 = 20. The soil properties represent a simplified single layer profile loosely based on parameters for the Bothkennar soft clay test site (e.g. Leroueil *et al.* [39] , Nash *et al.* [40]) proposed by Killeen and McCabe [35]. The stiff crust has been excluded from the soil profile. The values of E_{oed}^{ref}, E_{50}^{ref}, and E_{ur}^{ref} for the soil have been doubled and quadrupled for modular ratios of E_c/E_s = 10 and 5 respectively, while they have been halved for E_c/E_s = 40 (with all remaining soil properties remaining fixed), e.g. for a modular ratio of 40, $E_{oed,c}/E_{oed,s}$ = 56,858/1,421 = 40 at p^{ref} = 50 kPa.

It should be noted that the E_c/E_s values quoted here are just approximate indicators of the values that are actually modelled in the numerical model (such values can only be quoted as exact for a linear elastic soil model). In this case (for the HS Model), the soil stiffness depends on stress-level and over-consolidation ratio, so the values of E_c/E_s will only be exact for a normally consolidated soil for which the reference pressures in the soil and column materials are identical (in this case, at p^{ref} = 50 kPa).

Nash *et al.* [40], among others, have carried out extensive site characterisation at the Bothkennar site for which an overconsolidation ratio of between 1.5 and 1.6 has been reported for the lower Carse clay. However, since the analytical formulations consider no over-consolidation effect, an OCR of 1.0 was deemed more appropriate for defining the initial stress state for the subsequent numerical analyses (a fairer basis of comparsion). It is acknowledged that all soft clays will display at least a small overconsolidation effect, for example due to ageing, e.g. Degago [41], or groundwater level fluctuations As a check on the output, additional analyses using an OCR of 1.5 have been carried out to establish the influence of OCR on the results and it has subsequently been verified that the results are in fact unaffected.

4. Results

4.1 FE Predictions versus Field Data

The FE predictions for the different modular ratios have been put into general context by comparison with the field data from the database compiled by McCabe *et al.* [7], see Figure 6. The field data presented by McCabe *et* *al.* [7] pertain to long-term settlements from full-scale load tests and construction projects (both published and unpublished data). Predictions are plotted in terms of the reciprocal area-replacement ratio, A/A_c. McCabe *et al.* [7] have shown that Priebe's n_0 [13] produces a good match to the field data.

It should be recognised that comparison with the field data could be influenced by a number of factors, e.g. differences between 'as-constructed' column spacings and diameters, the stage after loading at which treated and untreated settlements are measured in the field (since columns accelerate primary consolidation, fair comparison with untreated soils will only be achieved if it is ensured that settlements are compared at the same stage of settlement, e.g. after primary consolidation, rather than at specific times). It also needs to be highlighted that the field data can only be plotted as a function of the area-replacement ratio because the corresponding modular ratio is unknown. Nevertheless, the comparison between the field data and the numerical predictions appears to be relatively good, which gives general confidence in the modelling procedure employed in this study.

The results in Figure 6 indicate that improvement factors predicted using the FE method increase as the modular ratio increases, which is to be expected. The FE n values appear to be converging as the modular ratio is increasing, i.e. the influence of the modular ratio becomes negligible (this is to be expected - only elastic design methods will place dependence on the modular ratio once the column has yielded and this is why elastic methods over-predict n values for high modular ratios). Parameters with a more dominant influence on the settlement behaviour include the friction angle of the column material, ϕ'_c, and the coefficient of lateral earth pressure, K.

4.2 Design Method Predictions versus FE Results (Base Case)

Settlement improvement factors calculated using design methods based on the unit cell assumption are compared to the numerical results in Figures 7a-d for the 'base case' (p_a = 100 kPa, φ'_c = 45°, ψ_c = 15°, K_0 = 0.44). Predictions are plotted as n/n_{PLAXIS} rather than n directly, e.g. $n/n_{PLAXIS} > 1$ indicates that the design method 'overpredicts' the settlement improvement factor (compared to the FE analyses), etc. It should be noted that in some cases, some of the analytical predictions are out of the range of plotted n/n_{PLAXIS} values and hence not every solution appears on every plot. The predictions using Aboshi *et al.* [10] have been obtained based on

SCFs (the SCF at the surface) calculated using the numerical output. Obviously this is a peculiar ‘design method’ which would not exist in reality. In general, the *n* value obtained using a numerical analysis will be used directly, rather than using the SCF to back-figure the *n* value. However, these predictions are just used to establish whether the simple equilibrium method can in fact be used to obtain reliable *n* values if sufficiently accurate input SCFs can be established.

Examination of Figures 7a-d indicates:

- It appears that the newest methods (i.e. Castro and Sagaseta [20], Pulko *et al.* [21]) offer the best agreement with the FE data over the entire range of modular ratios considered, i.e. $0.9 < n/n_{PLAXIS} < 1.1$. The predictions are in almost perfect agreement with the numerical predictions, and each other, despite the fact that the former is based on an undrained loading situation with subsequent consolidation while the latter is based on drained conditions. However, as highlighted in section 2.3, these methods (despite the different stress paths) are expected to give more or less identical results (the drained solution which considers the elastic strains in the column during its plastic deformation will produce the same results as the undrained plus consolidation solution). It is also worth noting that Balaam and Booker [15] will produce similar results. However, this method requires both numerical implementation and an iterative solution technique and as such has not been included in the graphs.
- In general, it appears that the agreement between the FE predictions (HS Model) and the elastic-plastic analytical predictions is improving a little at higher modular ratios ($1.0 < n/n_{PLAXIS} < 1.3$, e.g. Figures 7c and 7d). For Priebe [13,17], the reason for this could be the assumption of a significant bulging mechanism which is more prevalent in soft soils (e.g. CCE theory has been used by Hughes and Withers [28] to model the lateral bulging failure of a single column and hence predict its ultimate bearing capacity while Priebe [13] has also used CCE theory as the basis for the aforementioned design method). However, for Castro and Sagaseta [20] and Pulko *et al.* [21], the reason for the better predictions at higher modular ratios is more likely due to the variability of soil stiffness with stress level. The analytical formulations assume a constant stiffness modulus for the soil and column. However, the HS Model in PLAXIS accounts for the stress dependency of stiffness (i.e. the stiffness depends on the confining pressure, σ'_3, e.g. Equation 6). For low modular ratios, the column will not take as much of the load as it would take for higher modular ratios, i.e. for a lower modular ratio, the value of σ'_3 in the column will be lower. Accordingly, the value of σ'_3 in the soil will be higher at lower modular ratios than at higher modular ratios. In general, the differences between

the analytical and FE predictions will be more evident in situations where elastic strains are more important (e.g. low A/A_c values)."

- It is very noticeable that the majority of elastic-plastic methods appear to converge ($1.0 < n/n_{PLAXIS} < 1.3$) as the modular ratio increases (more realistic for soft soils, e.g. Figures 7c and 7d), highlighting the fact that regardless of the basis or corresponding assumptions made in the derivation of each method, predicted settlement improvement factors are in the same range.
- Elastic methods, e.g. Balaam and Booker [11], overpredict the settlement improvement for large modular ratios, i.e. $n/n_{PLAXIS} >> 1.4$ for modular ratios of 20 and 40 (Figures 7c and 7d respectively). For elastic methods, the SCF will be too high because yielding of the column material is ignored (yielding/plastic strains reduces the SCF and hence the predicted settlement improvement).
- Priebe's n_0 [13] is independent of the modular ratio, E_c/E_s (n_0 predictions are closer to the FE results as the modular ratio increases because the FE n values rise and thus n/n_{PLAXIS} approaches 1).
- Priebe's n_1 [17] predicts less of an improvement than n_0 in all cases, i.e. accounting for the compressibility of the column material removes any overestimation of the settlement improvement predicted by n_0. For lower A/A_c values (i.e. more stone), there is more compressible column material to be accounted for, and hence n_1 gets further and further away from n_0 as the area-replacement ratio increases (lower A/A_c values).
- n_2/n_{PLAXIS} (more lateral support) is above n_1/n_{PLAXIS} in all four graphs. The difference between n_2/n_{PLAXIS} and n_1/n_{PLAXIS} would be more pronounced for a higher at-rest coefficient of lateral earth pressure, K, e.g. for K = 1, n_2/n_{PLAXIS} would be above n_0/n_{PLAXIS} in some cases.
- Pulko and Majes [18] appears to predict n/n_{PLAXIS} values consistently in the range 1.1 to 1.4 for modular ratios of 10, 20 and 40. This clearly shows how neglecting the elastic strains in the column during its plastic deformation for a drained solution influences the results (i.e. over-predicts settlement improvement factors because neglecting the elastic strains means lower 'treated' settlements are predicted). As is clear from Figure 7, the deviation from $n/n_{PLAXIS} = 1$ is larger at low A/A_c values, i.e. in cases where the elastic strains are more important.
- Borges *et al.* [19] have developed a design chart based on Equation 5. The design chart indicates that the design equation should perhaps only be applied in a certain range (although not explicitly stated in the paper). It appears that the design equation predicts much less of an improvement than the other design methods for modular ratios of 5, 10 and 20 ($n/n_{PLAXIS} < 0.8$), i.e. n values < 1.5 (which do not appear on the design chart). For $E_c/E_s = 40$, Borges *et al.* [19] shows better agreement with the other design methods. For

ever-increasing modular ratios, the method proposed by Borges *et al.* [19] predicts ever-increasing improvement factors (greater than those predicted by the analytical methods), so it appears the method is considerably more sensitive to the modular ratio than the analytical design methods (owing to the numerical basis of the method).

- The simple equilibrium method described by Aboshi *et al.* [10], based on PLAXIS-calculated surface SCFs (see Figure 8), consistently predicts $n/n_{PLAXIS} \approx 0.9$ irrespective of the modular ratio or area-replacement ratio (i.e. conservative design predictions, $n/n_{PLAXIS} < 1$). This indicates that the method, despite its simple nature, could be safely applied in real-life design situations provided that the SCF is not over-estimated. Note that if the average SCF over the complete soil profile was used instead of the SCF at the surface, n/n_{PLAXIS} would be marginally lower for each modular ratio ($n/n_{PLAXIS} \approx 0.8$), i.e. $SCF_{AVERAGE} < SCF_{SURFACE}$.

4.3 Parameter Sensitivity Study

The comparisons carried out in the previous section clearly indicate that the methods derived by Castro and Sagaseta [20] and Pulko *et al.* [21] offer the best agreement with finite element predictions for the 'base case' considered. Based on this, a parameter sensitivity study is carried out to establish the effect of altering selected parameters (p_a, φ'_c, ψ_c, K_0). In addition, the influence of these parameters on Priebe's n_2 [17] has also been examined because of its popularity in European geotechnical practice.

4.3.1 Load Level (p_a)

The behaviour of the composite soil-column system has also been studied under 50 kPa and 75 kPa loads (with all other parameters fixed). As before, design method predictions have been compared to FE results (Figures 9-10). The elastic-plastic design methods predict larger improvement factors when columns are subjected to lower applied loads (as does PLAXIS), indicating that stone columns are more effective at lower load levels (less yielding). Elastic design methods have no dependency on load level (e.g. Balaam and Booker [11]), nor does Priebe's n_0 [13] or the FE-based method derived by Borges *et al.* [19] which depends only on A_c/A and E_c/E_s. The SCFs used to obtain n values for Aboshi *et al.* [10] have again been obtained from the FE output.

As was the case with p_a = 100 kPa, it is worth noting that the elastic-plastic method improvement factors converge with increasing modular ratio for both p_a = 50 kPa (e.g Figures 9c and 9d) and 75 kPa (e.g. Figures 10c and 10d), i.e. 1.0 < n/n_{PLAXIS} < 1.3 (despite some divergence for large quantities of stone, e.g. A/A_c < 4). For E_c/E_s = 5, Pulko and Majes [18] predicts lower *n* values at lower applied loads (this is in contrast with other methods, e.g. Priebe [17], Castro and Sagaseta [20], Pulko *et al.* [21]), and perhaps indicates that the method may not be applicable for $E_c/E_s \leq 5$. The reason for the discrepancy at E_c/E_s = 5 has been discussed earlier. For low modular ratios, the elastic strains in the column during its plastic deformation have a significant influence (i.e. because the elastic stiffness of the column is of the same order of that of the soil) and cannot be neglected when adopting a drained approach. It is because of such extreme cases (and also for realistic values for encased stone columns) that Pulko *et al.* [21] improved on the earlier solution by Pulko and Majes [18].

Load level affects the depth to which plastic strains appear in the column (yielding depends on the dimensionless load factor $p_a/(\gamma'.z)$ where γ' is the soil unit weight and *z* is the depth below ground level), i.e. yielding starts at the surface and progresses downwards with time (Castro and Sagaseta [20]); higher loads result in more and more column yielding. Yielding has been confirmed in the FE analyses by examining plots of 'plastic points' (plastic points are the stress points in the FE model that are in a plastic state, e.g. stresses lying on the Mohr-Coulomb failure surface or on the shear hardening envelope, denoted by red and green cubes in the PLAXIS output program respectively). The red Mohr-Coulomb points have been used to affirm the presence of yielding. Despite the different stress paths (drained versus undrained conditions) used by Castro and Sagaseta [20] and Pulko *et al.* [21], these methods result in *n* values that are in almost perfect agreement with one another, and under both the 50 kPa and 75 kPa loads, their predictions are consistently in best agreement with the FE results, regardless of the modular ratio or column spacing (i.e. n/n_{PLAXIS} is almost always in the range 0.9-1.1 which gives considerable confidence in these design methods).

4.3.2 Friction Angle of Column Material (φ'_c)

Priebe [13,17], Pulko and Majes [18], Castro and Sagaseta [20], and Pulko *et al.* [21] predict larger *n* values for higher column friction angles, φ'_c (with the exception of Pulko and Majes [18] at E_c/E_s = 5, again illustrating that the method may not be applicable for $E_c/E_s \leq 5$) The method by Borges *et al.* [19] is independent of the friction angle of the column material, while the elastic methods are over-simplified in this respect. The influence of the friction angle (φ'_c = 35º, 40º, 45º) of the granular material is clearly evident on the n/n_{PLAXIS} values predicted by the favoured analytical settlement design methods in Figures 11a-d for the four different modular ratios considered (the other parameters have been fixed at p_a = 100 kPa, ψ_c = 15º and K_0 = 0.44). n_{PLAXIS} has been plotted against A/A_c in Figures 12a-d in order to show how the FE n values are influenced by the friction angle of the granular material.

- As would be expected, lower friction angles result in lower improvement factors, e.g. Figures 12a-d.
- Priebe's n_2 [17] appears to consistently overpredict n values (i.e. $n/n_{PLAXIS} > 1$) for all friction angles considered in this study.
- The agreement between Priebe's n_2 [17] and the other analytical predictions is better for lower friction angles (e.g. φ'_c = 35º, e.g. $n/n_{PLAXIS} \approx 1.1$) than it is for higher friction angles (φ'_c = 45º, e.g. $n/n_{PLAXIS} \approx 1.3$). This is generally why Priebe's [17] method tends to be used with conservative estimates for the friction angle of the granular column material.
- Predicted n values from Castro and Sagaseta [20] and Pulko *et al.* [21] are in almost perfect agreement with one another for all modular ratios and friction angles considered (and comparison with the FE output is again excellent, i.e. $0.9 < n/n_{PLAXIS} < 1.1$).
- Their predictions appear to be in better agreement with Priebe's n_2 [17] as the modular ratio increases (i.e. softer soils with more associated bulging).

Examination of predicted SCFs (Figure 13) illustrates part of the reason for the considerably different n value predictions for the design methods.

- Predicted SCFs are in excellent agreement for Castro and Sagaseta [20] and Pulko *et al.* [21] with ever so slight differences apparent for closely spaced columns ($A/A_c < 4$). When the columns are closely spaced, the elastic strains of the column have a greater influence and this is the reason for the slight differences in the SCFs.
- Priebe's [17] predicted SCFs are noticeably higher than these predictions. However, it is not appropriate to use Priebe's [17] method to estimate SCFs because the method merely uses the SCF as a post-correction to work out n_2. The SCF is thus not considered a result of the method.

- The SCFs calculated using PLAXIS 2D are in the range predicted by Castro and Sagaseta [20] and Pulko *et al.* [21] which highlights why the predicted n values are also in the same range.
- n values and SCFs are directly related for analytical methods but this is not the case for Priebe's [17] method because of its empirical basis. Priebe's [17] method is much better at predicting n than it is at predicting SCFs (it is not commonly used to predict SCFs). As the post-correction of the column stiffness is carried out independently of the initial stresses (which are used as the basis for working out SCFs where analytical methods are concerned), Priebe's [17] method does not consider the elastic modulus of the column to predict the SCF.
- Differences between the predicted SCFs are most evident for the lowest modular ratio (E_c/E_s = 5, e.g. Figure 12a). The corresponding improvement factors also exhibit the largest differences for this case (Figure 11a).
- The good agreement between PLAXIS-calculated n values and SCFs with those predicted by Castro and Sagaseta [20] and Pulko *et al.* [21] again affirms their greater applicability in design.

4.3.3 Dilatancy Angle of Column Material (ψ_c)

Pulko and Majes [18], Castro and Sagaseta [20], and Pulko *et al.* [21] predict larger n values for higher dilatancy angles, ψ_c. The n values predicted by elastic methods (e.g. Balaam and Booker [11]) and Borges *et al.* [19] are independent of the dilatancy angle. The influence of the dilatancy angle (Figures 14a-d) of the granular material has been examined in the range $0^o < \psi_c < 15^o$ for Castro and Sagaseta [20] and Pulko *et al.* [21]. In this case, the remaining parameters have been fixed at those corresponding to the base case (p_a = 100 kPa, $\varphi'_c = 45^o$, K_0 = 0.44). Priebe's [17] method has been formulated on the assumption of constant volume deformation during yield, i.e. $\psi_c = 0^o$. Based on this, it would be expected that Priebe's n_2 [17] would be in direct agreement with Castro and Sagaseta [20] and Pulko *et al.* [21] for $\psi_c = 0^o$. The n_{PLAXIS} predictions have been included in Figures 15a-d in order to show the direct influence of ψ_c on n (higher ψ_c values lead to higher n values). Examination of Figures 14a-d indicates:

- Priebe n_2 [17] tends to significantly over-predict settlement improvement factors in all cases for a column that does not exhibit dilatant behaviour (i.e. n/n_{PLAXIS} > 1.4). It thus appears that the method is more applicable for dilatant columns (i.e. larger n values) even though it has been formulated (with empiricism) for non-dilatant column material. Note that the comparisons in section 4.2 were with FE analyses for which $\psi_c = 15^o$.

- The settlement improvement factors predicted by the newer methods are again in direct agreement with one another for all cases considered and their agreement with HS Model n values is particularly good for all modular ratios (i.e. $1.0 < n/n_{PLAXIS} < 1.1$ with slight departures evident for $A/A_c < 4$).
- Focusing on the predicted SCFs (Figures 16a-d), similar conclusions as were drawn with regard the friction angle can again be drawn. The HS Model SCFs are in almost direct agreement with the SCFs predicted by Castro and Sagaseta [20] and Pulko *et al.* [21].

4.3.4 Coefficient of Lateral Earth Pressure (*K*)

Priebe [17], Pulko and Majes [18], Castro and Sagaseta [20], and Pulko *et al.* [21] predict larger n values for higher K values (i.e. more lateral support). The n values predicted by elastic methods (e.g. Balaam and Booker [11]) and Borges *et al.* [19] are independent of K. The sensitivity of Priebe [17], Castro and Sagaseta [20] and Pulko *et al.* [21] with respect to the coefficient of lateral earth pressure following column installation (K) has been examined for three different K values (K_0 = 0.44, 0.7, 1.0); these values have been chosen based on theoretical considerations mentioned in section 2.1. Predictions are plotted in Figures 17a-d. Note that K for untreated case (no columns) is maintained equal to the at-rest value ($K_0 = 1 - \sin \varphi' = 0.44$). Again, the direct influence of K on the FE n values is plotted in Figures 18a-d. Larger K values result in larger settlement improvement factors, i.e. larger K values leads to increased horizontal stresses in the soil, hence providing more resistance to lateral bulging of the granular material.

Similar conclusions to the previous sensitivity studies can again be drawn, i.e. predictions with the newer methods are in good comparison with one another but again, Priebe [17] over-predicts the improvement (although Priebe's [17] predictions are closer to the newer methods at the higher modular ratios, e.g. Figure 17d). SCFs (at the surface) predicted by Priebe [17], Pulko *et al.* [21] and Castro and Sagaseta [20] are independent of the value of K (Figure 19). FE-predicted SCFs are in good agreement with the newer analytical design methods.

5. Conclusions

- Elastic methods will overpredict the settlement improvement and should really only be used in relatively stiff soils in which the modular ratio, E_c/E_s, will be relatively small (or perhaps with unrealistic conservative low values of the modular ratio).
- Both the analytical and finite element predictions are in good agreement with the bottom feed field data for E_c/E_s = 10, 20, and 40.
- Based on the results, it is suggested that the newest methods (Castro and Sagaseta [20], Pulko *et al.* [21]) offer the most reliable predictions which tend to be consistently in excellent agreement with FE predictions for end-bearing columns (owing to the considerably more rigorous theoretical basis associated with the newer methods as opposed to the empirical correlations used to calculate Priebe's [17] n_1 and n_2).
- Analytical solutions assume the soil to behave in a linear elastic manner while in the numerical study carried out in this paper, the soil behaviour includes the stress dependency of stiffness (a more realistic assumption). This may lead to some differences between the analytical solutions and the numerical results, but as is evident from the results above, these differences are small for Castro and Sagaseta [20] and Pulko *et al.* [21].
- The parameter sensitivity study looking at load level (p_a), column friction angle (φ'_c), column dilatancy angle (ψ_c) and the coefficient of lateral earth pressure (K) has shown Priebe's n_2 [17] method to consistently over-predict improvement factors, which would suggest that some caution should be applied when applying it in practice (which already seems to be the case in fact because the method tends to be used with conservative values for the column friction angle).
- Predicted SCFs by both Castro and Sagaseta [20] and Pulko *et al.* [21] are consistently in excellent agreement with those predicted by the HS Model.
- Priebe's [17] method should not be used to calculate SCFs (the SCF is merely used as a post-correction to work out n_2).
- In summary, it is proposed that the design methods derived by Castro and Sagaseta [20] and Pulko *et al.* [21] should be used more often in geotechnical practice because these methods give more realistic results and allow for the consideration of significantly more input data.

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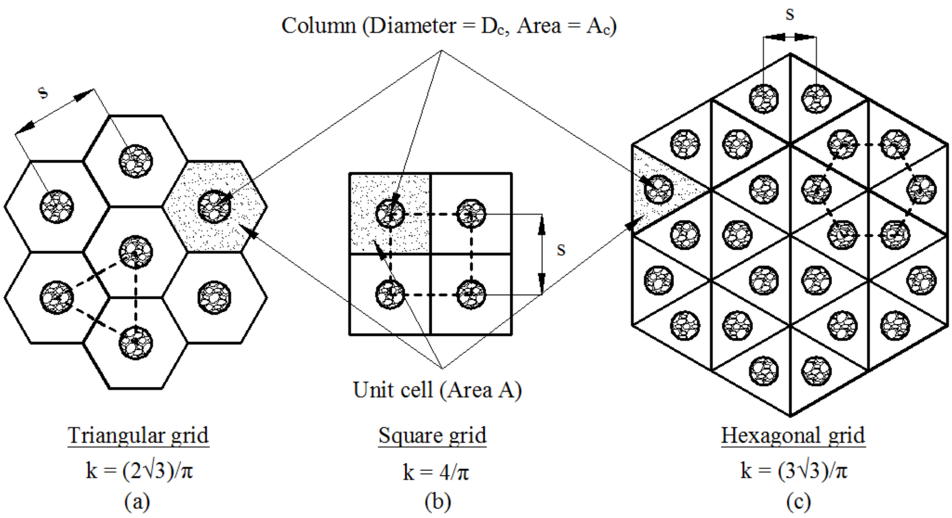


Fig. 1 Typical Column Grids encountered in practice; (a) triangular (b) square (c) hexagonal

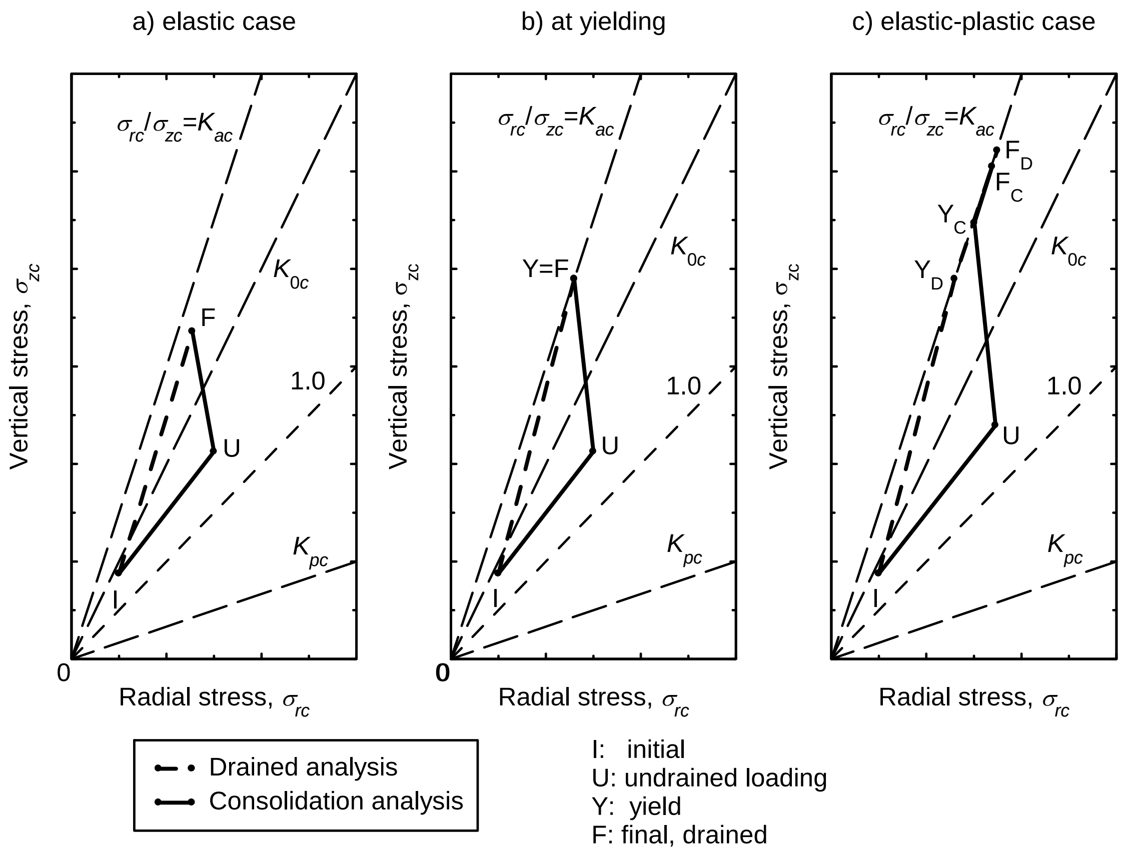


Fig. 2 Stress paths in the column; (a) elastic case (b) at yielding (c) elastic-plastic case

(Castro and Sagaseta [29])

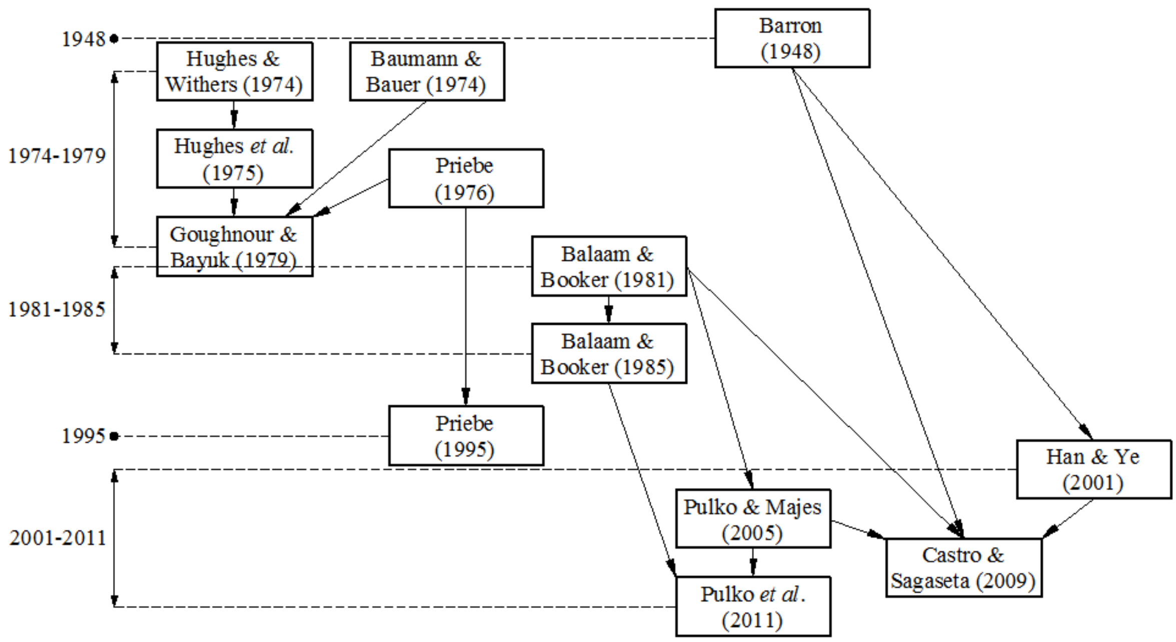


Fig. 3 Development of Settlement Prediction Methods (Unit Cell)

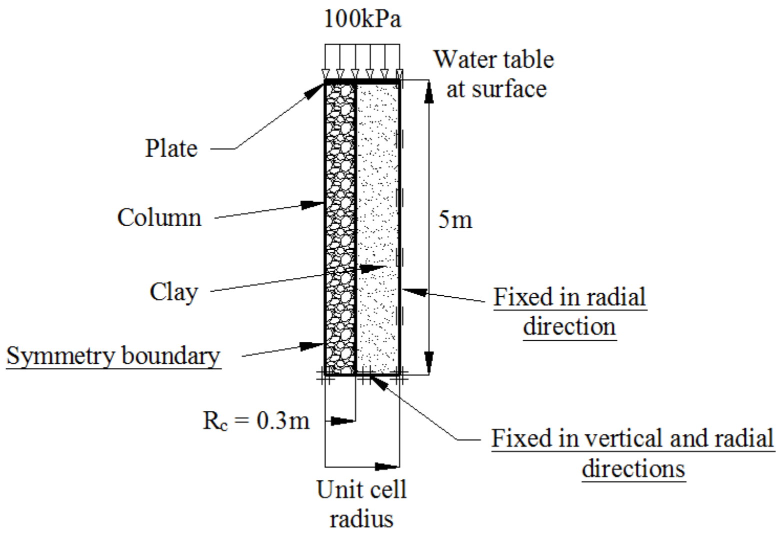
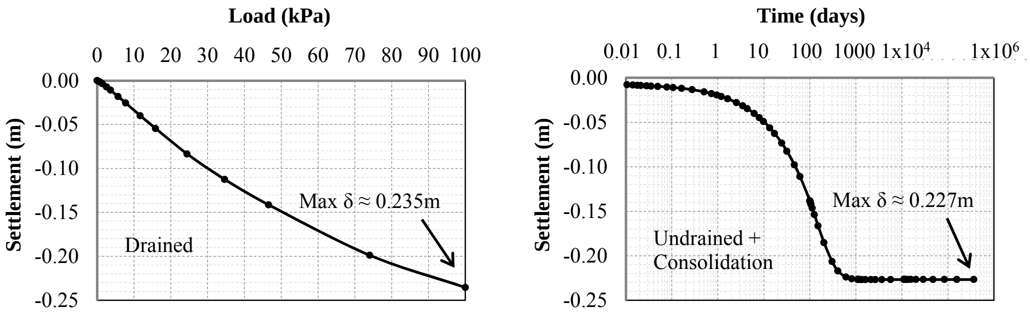
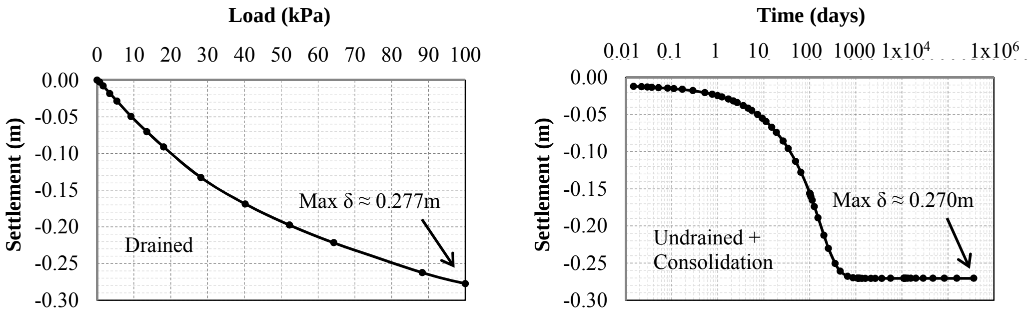


Fig. 4 Axisymmetric Unit Cell Model (100kPa Load)



(a) E_c/E_s = 20, K = 1.0 (No Columns - i.e. s_0)



(b) E_c/E_s = 20, K = 1 - sin φ' = 0.44 (No Columns - i.e. s_0)

Fig. 5 $\delta_{\text{Drained}} = \delta_{\text{Undrained + Consolidation}}$

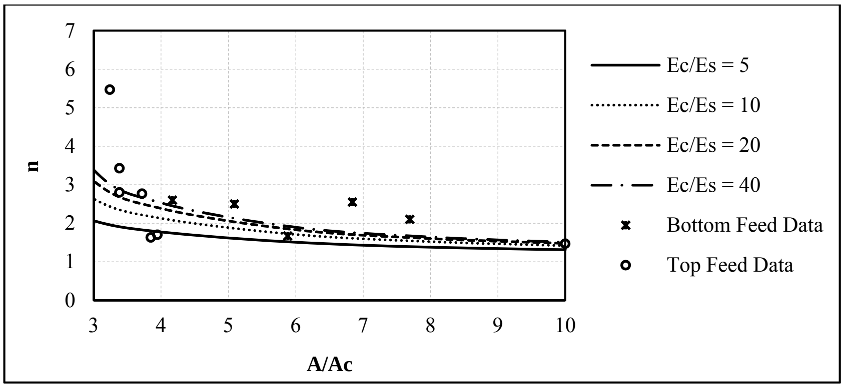


Fig. 6 FE Predictions versus Field Data

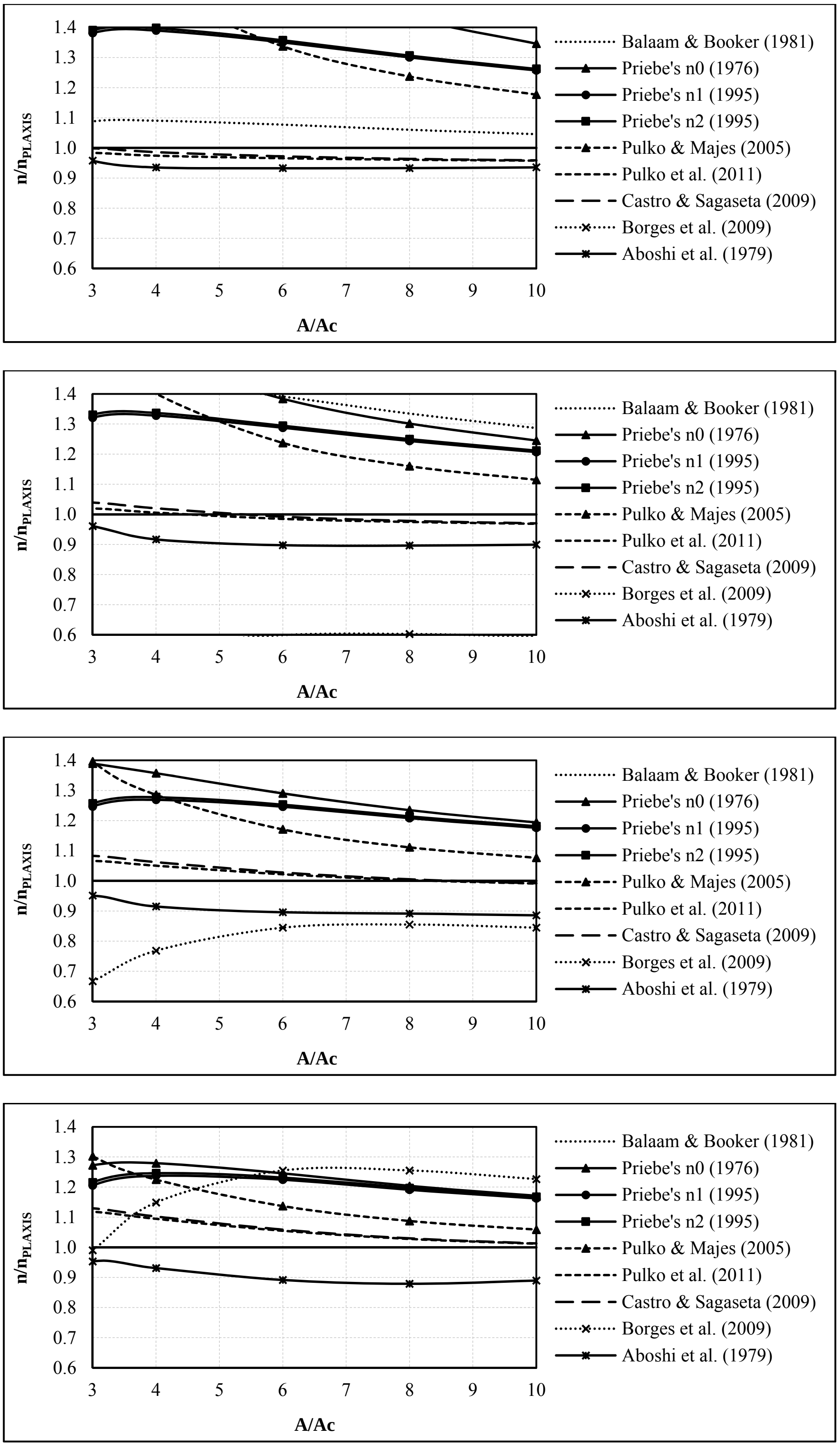


Fig. 7 n/n_{PLAXIS} versus A/A_c (p_a = 100kPa); (a) E_c/E_s = 5 (b) E_c/E_s = 10 (c) E_c/E_s = 20 (d) E_c/E_s = 40

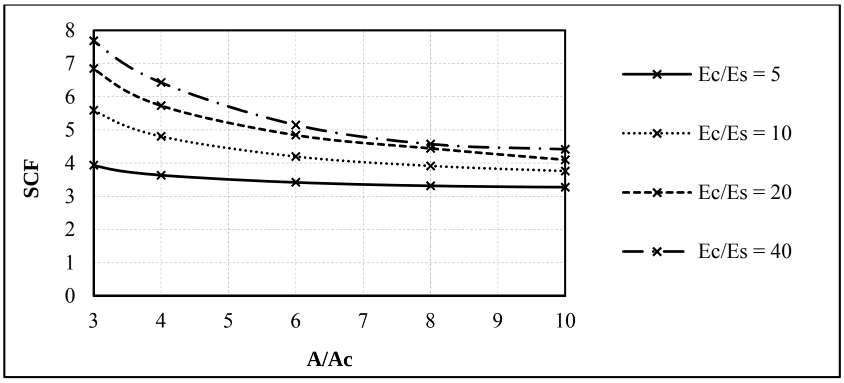


Fig. 8 PLAXIS-calculated SCFs versus A/A_c (Base Case)

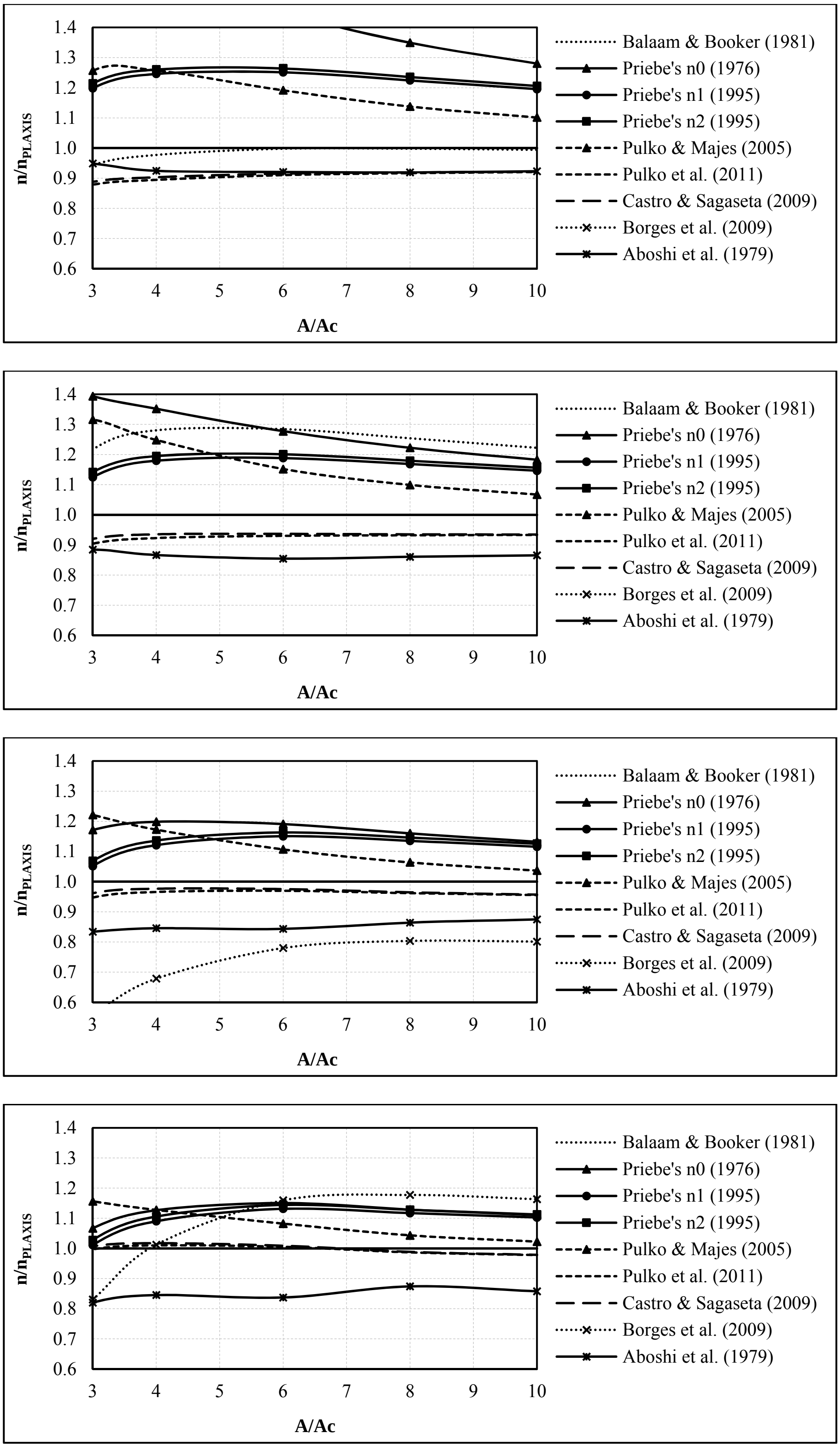


Fig. 9 n/n_{PLAXIS} versus A/A_c (p_a = 50kPa); (a) E_c/E_s = 5 (b) E_c/E_s = 10 (c) E_c/E_s = 20 (d) E_c/E_s = 40

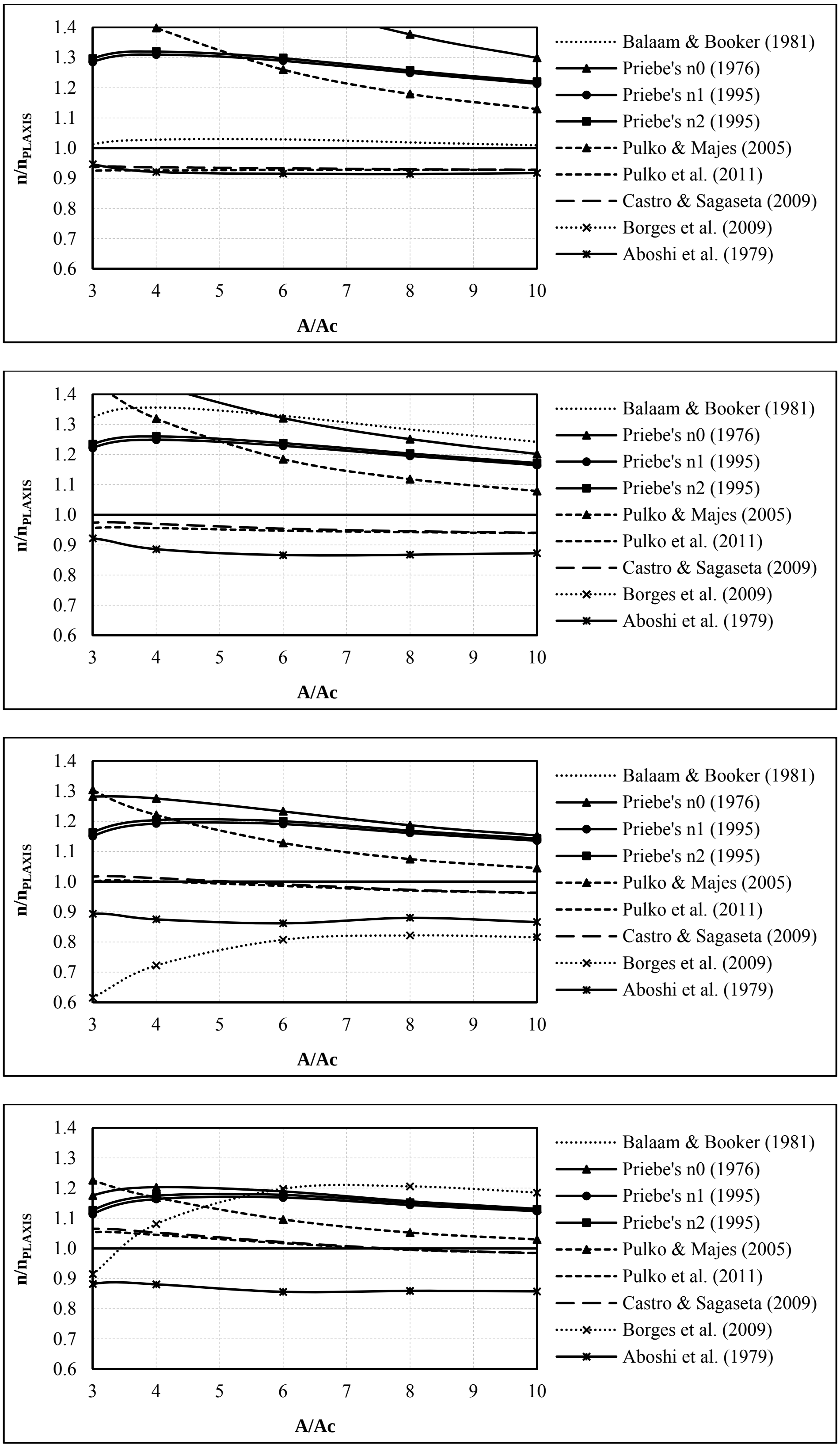


Fig. 10 n/n$_{PLAXIS}$ versus A/A$_c$ (p_a = 75kPa); (a) E_c/E_s = 5 (b) E_c/E_s = 10 (c) E_c/E_s = 20 (d) E_c/E_s = 40

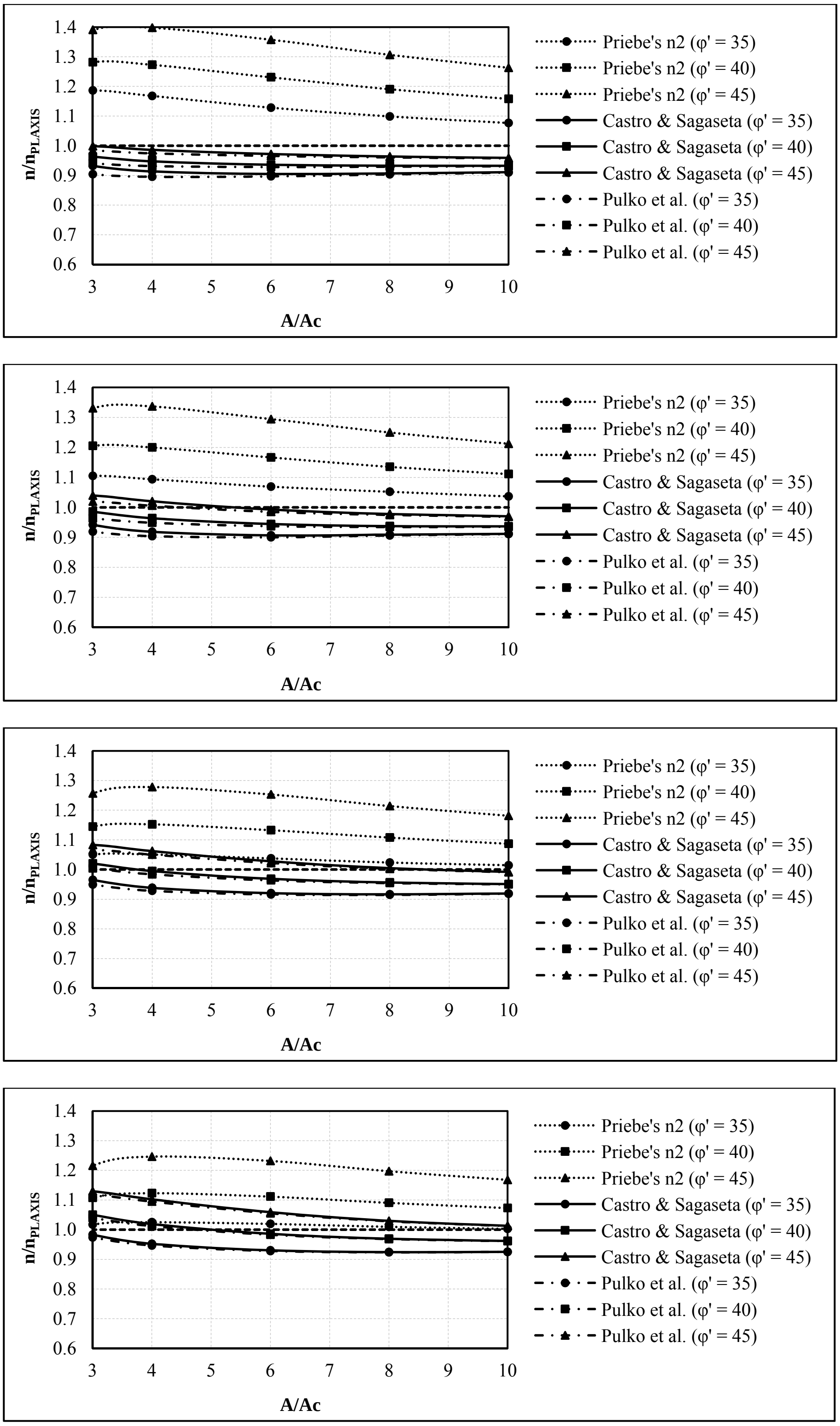


Fig. 11 n/n_{PLAXIS} versus A/A_c (influence of φ'_c); (a) $E_c/E_s = 5$ (b) $E_c/E_s = 10$ (c) $E_c/E_s = 20$ (d) $E_c/E_s = 40$

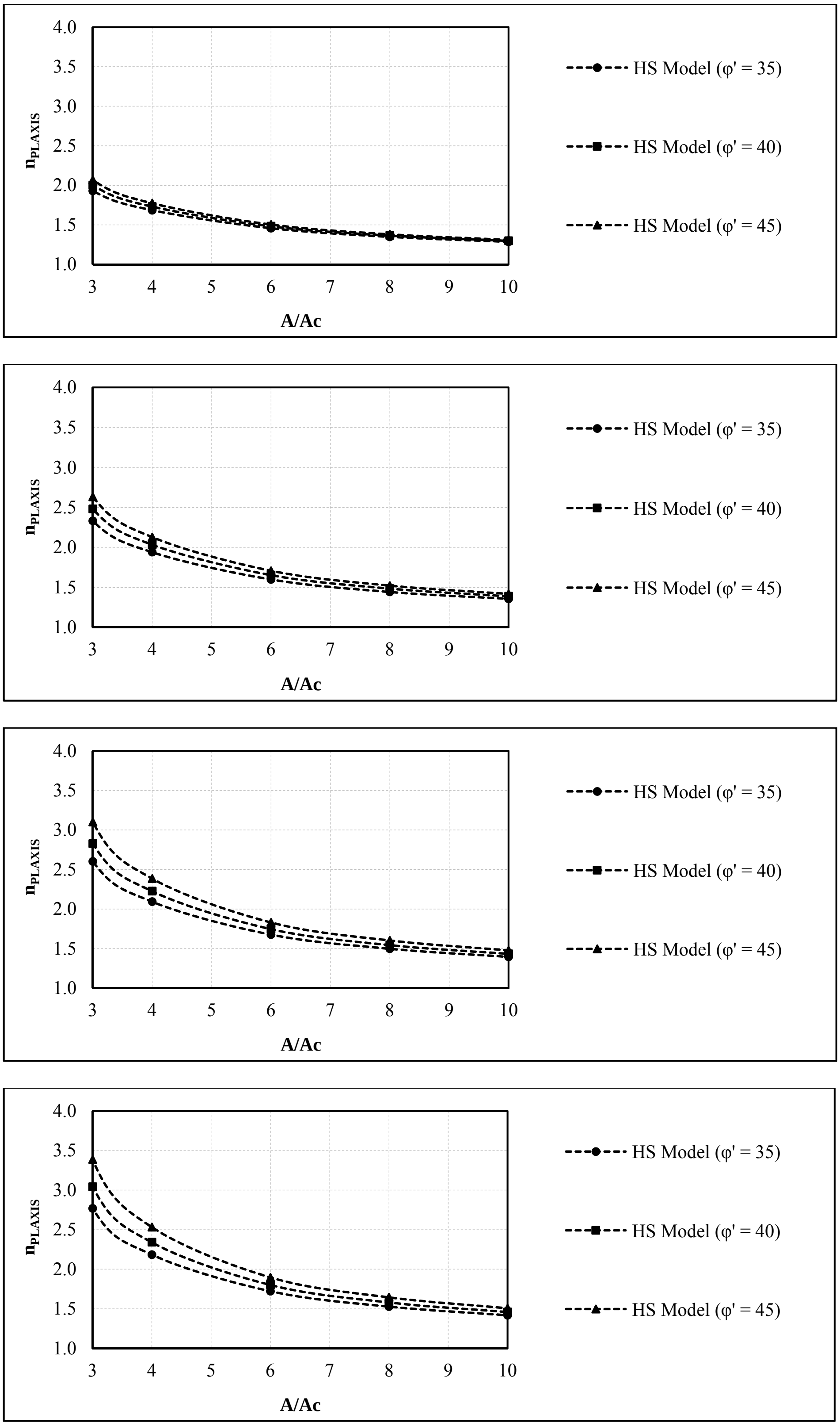


Fig. 12 n_{PLAXIS} versus A/A_c (influence of φ'_c); (a) $E_c/E_s = 5$ (b) $E_c/E_s = 10$ (c) $E_c/E_s = 20$ (d) $E_c/E_s = 40$

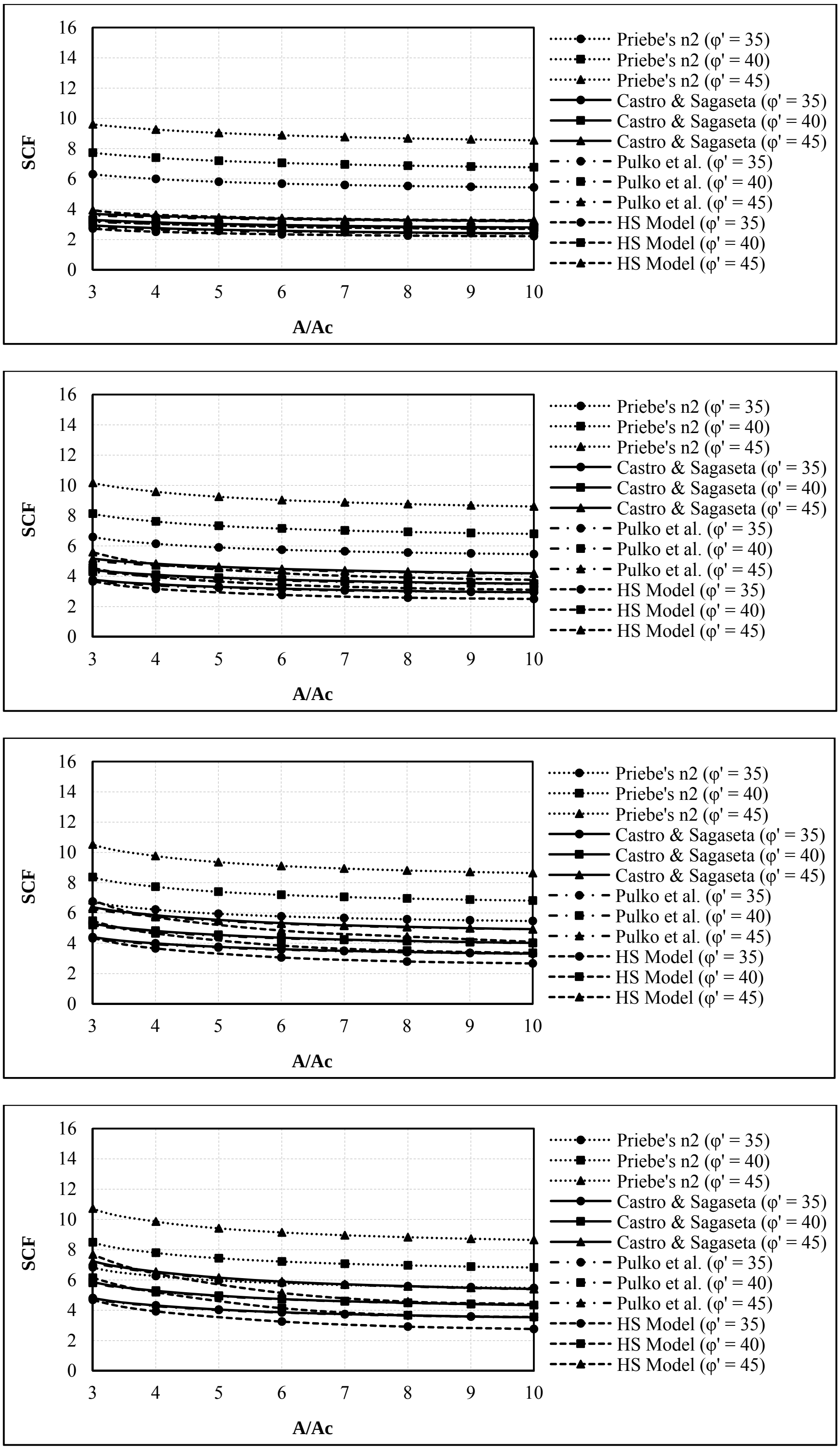


Fig. 13 SCF versus A/A_c (influence of φ'$_c$); (a) $E_c/E_s = 5$ (b) $E_c/E_s = 10$ (c) $E_c/E_s = 20$ (d) $E_c/E_s = 40$

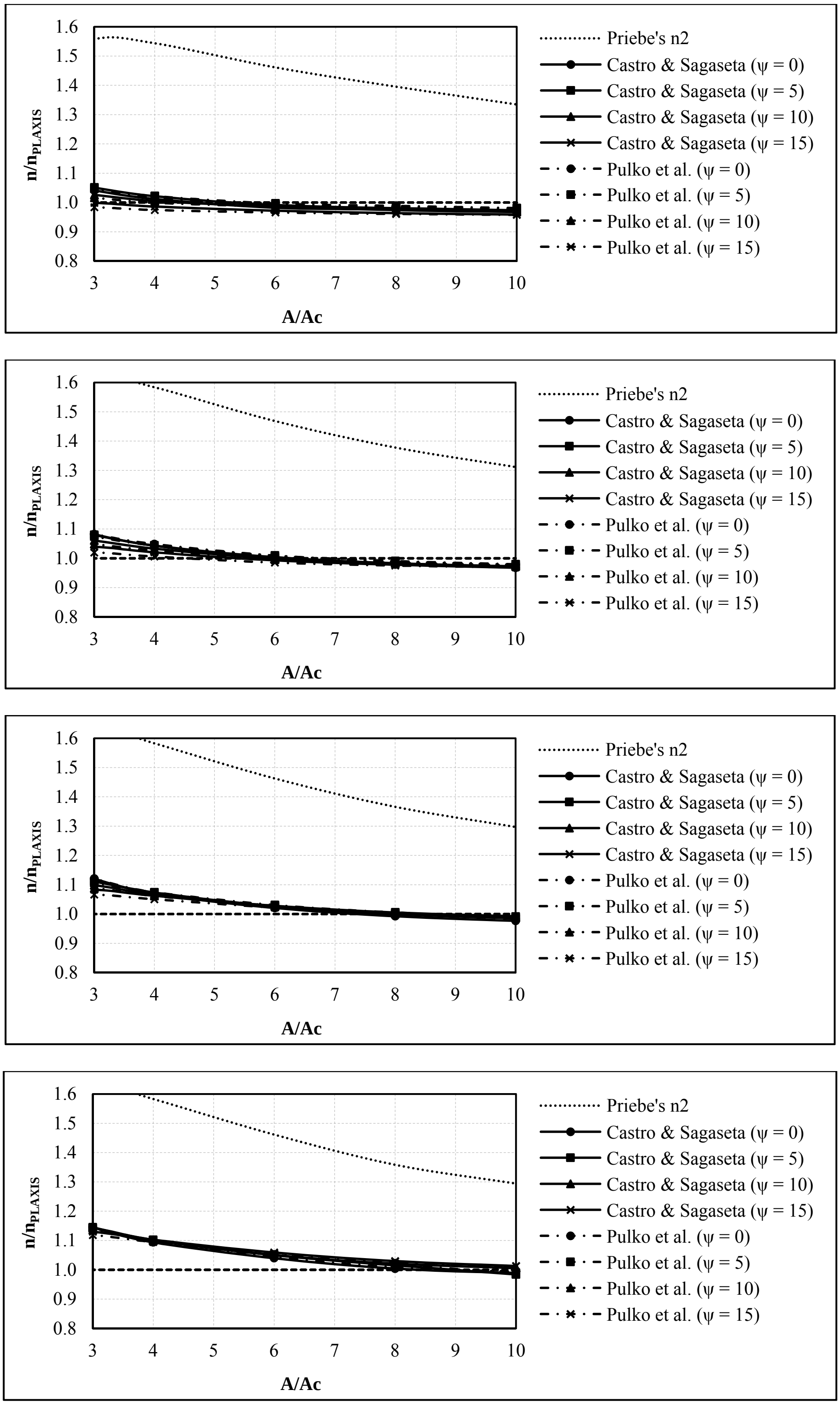


Fig. 14 n/n_{PLAXIS} versus A/A_c (influence of ψ_c); (a) $E_c/E_s = 5$ (b) $E_c/E_s = 10$ (c) $E_c/E_s = 20$ (d) $E_c/E_s = 40$

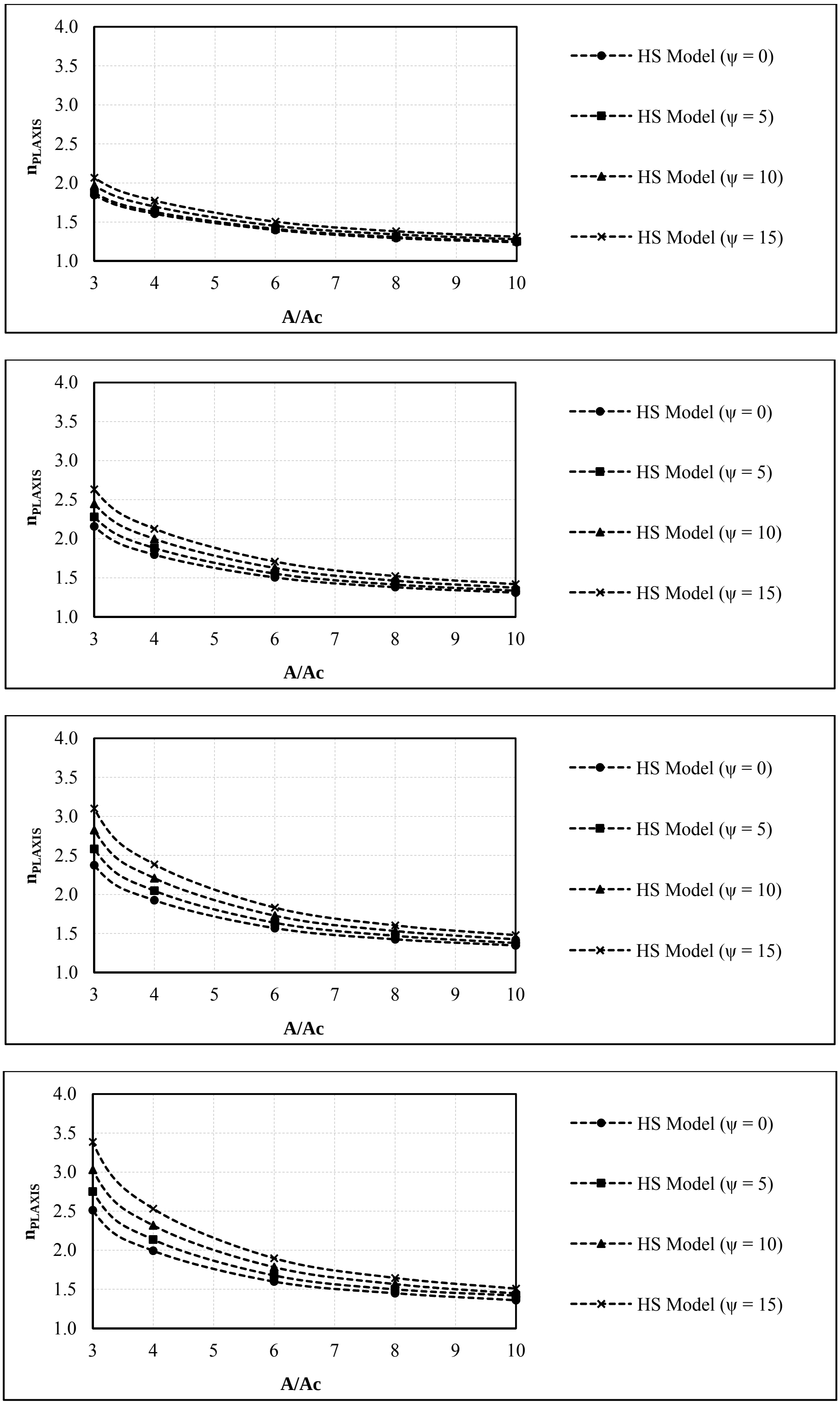


Fig. 15 n_{PLAXIS} versus A/A_c (influence of ψ_c); (a) $E_c/E_s = 5$ (b) $E_c/E_s = 10$ (c) $E_c/E_s = 20$ (d) $E_c/E_s = 40$

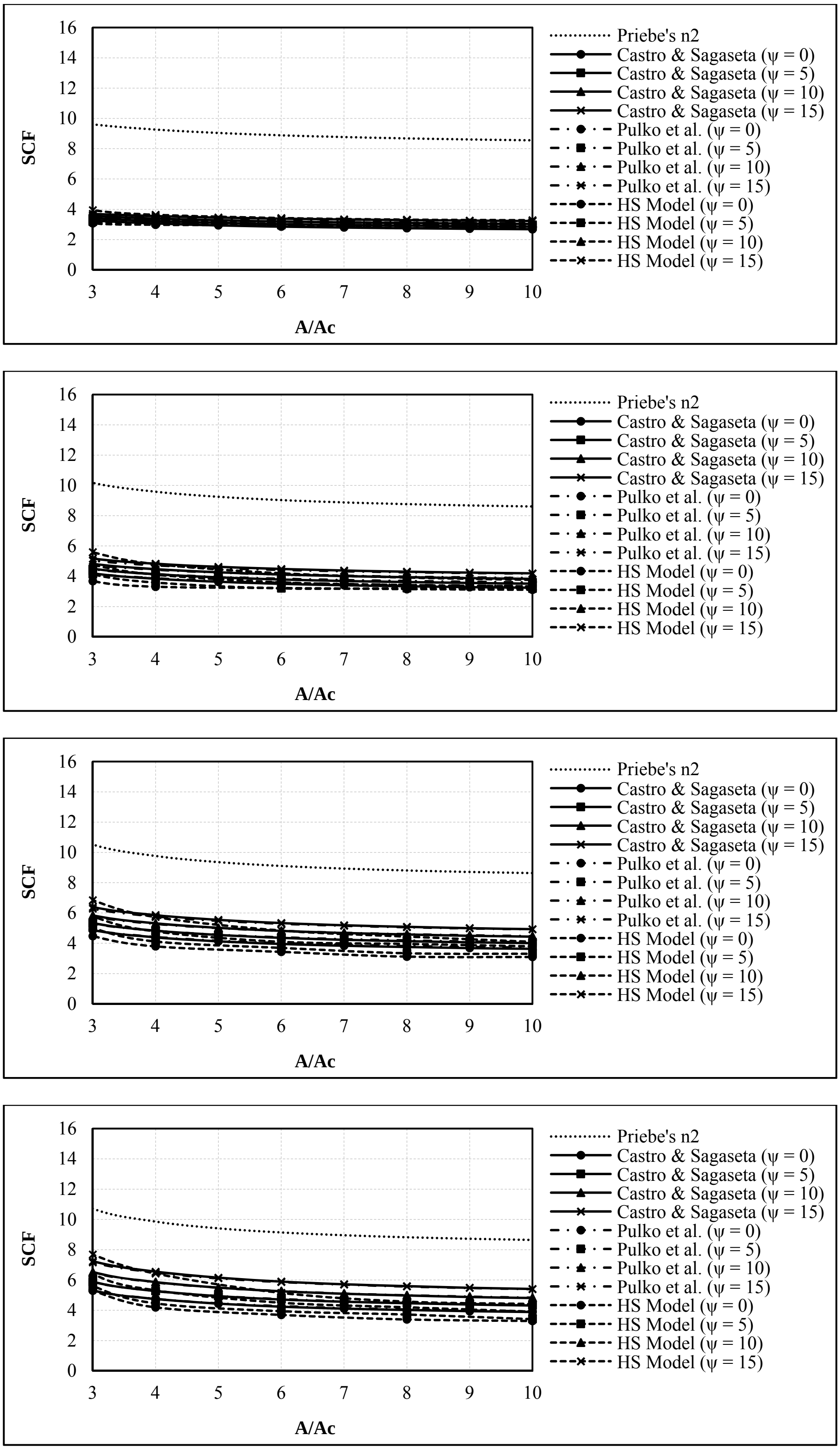


Fig. 16 SCF versus A/A_c (influence of ψ_c); (a) $E_c/E_s = 5$ (b) $E_c/E_s = 10$ (c) $E_c/E_s = 20$ (d) $E_c/E_s = 40$

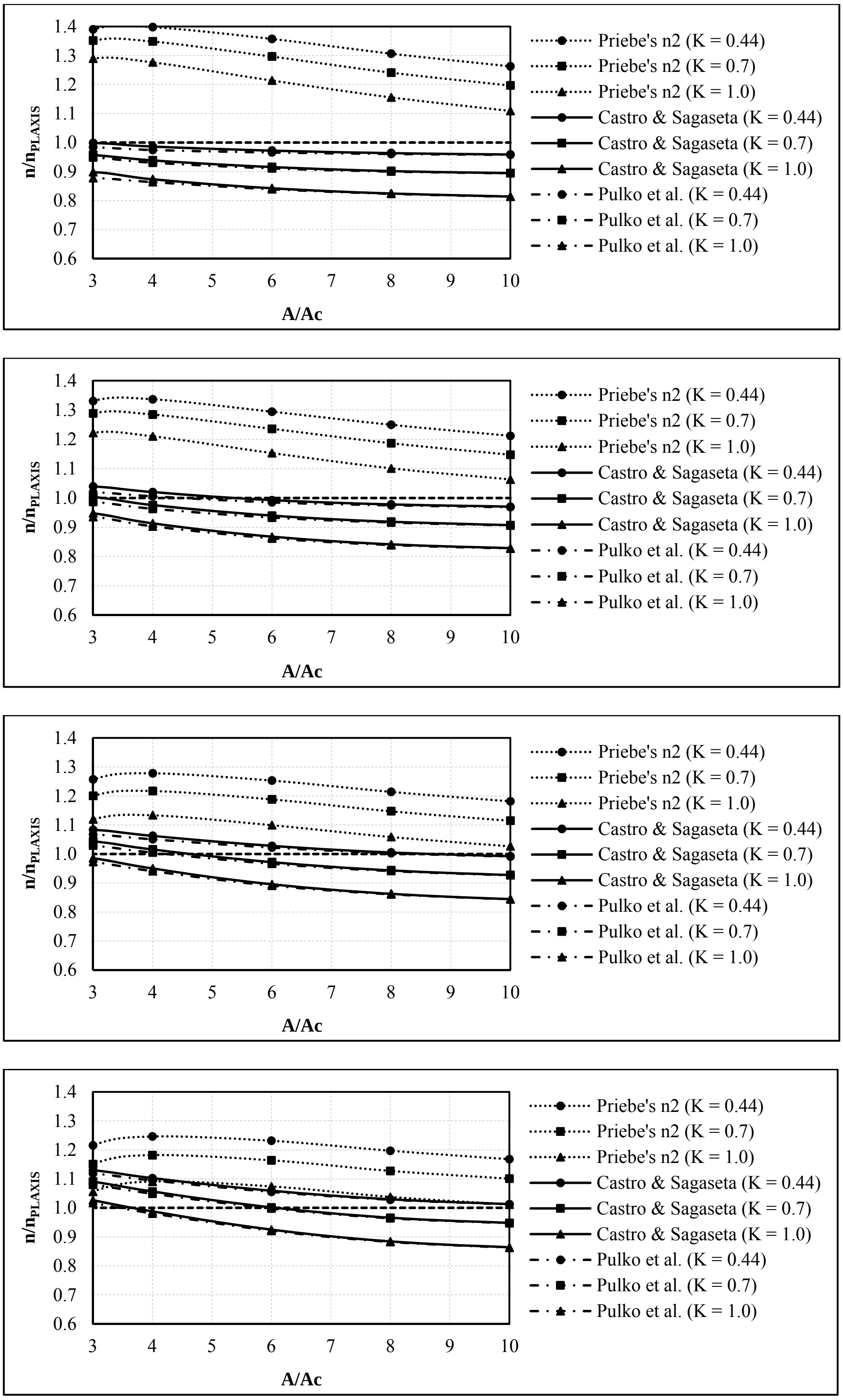


Fig. 17 n/n_{PLAXIS} versus A/A_c (influence of K_0); (a) $E_c/E_s = 5$ (b) $E_c/E_s = 10$ (c) $E_c/E_s = 20$ (d) $E_c/E_s = 40$

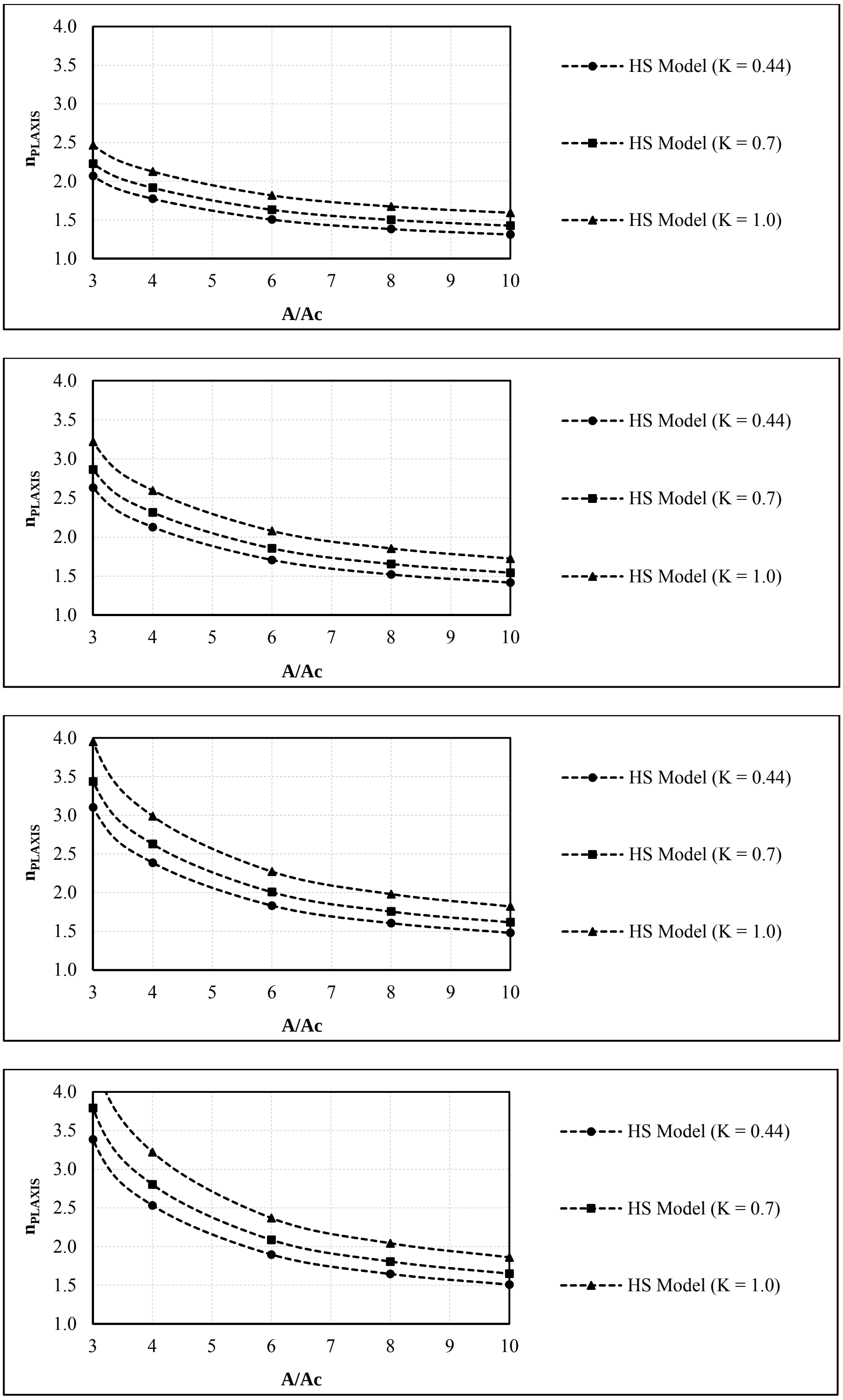


Fig. 18 n_{PLAXIS} versus A/A_c (influence of K_0); (a) $E_c/E_s = 5$ (b) $E_c/E_s = 10$ (c) $E_c/E_s = 20$ (d) $E_c/E_s = 40$

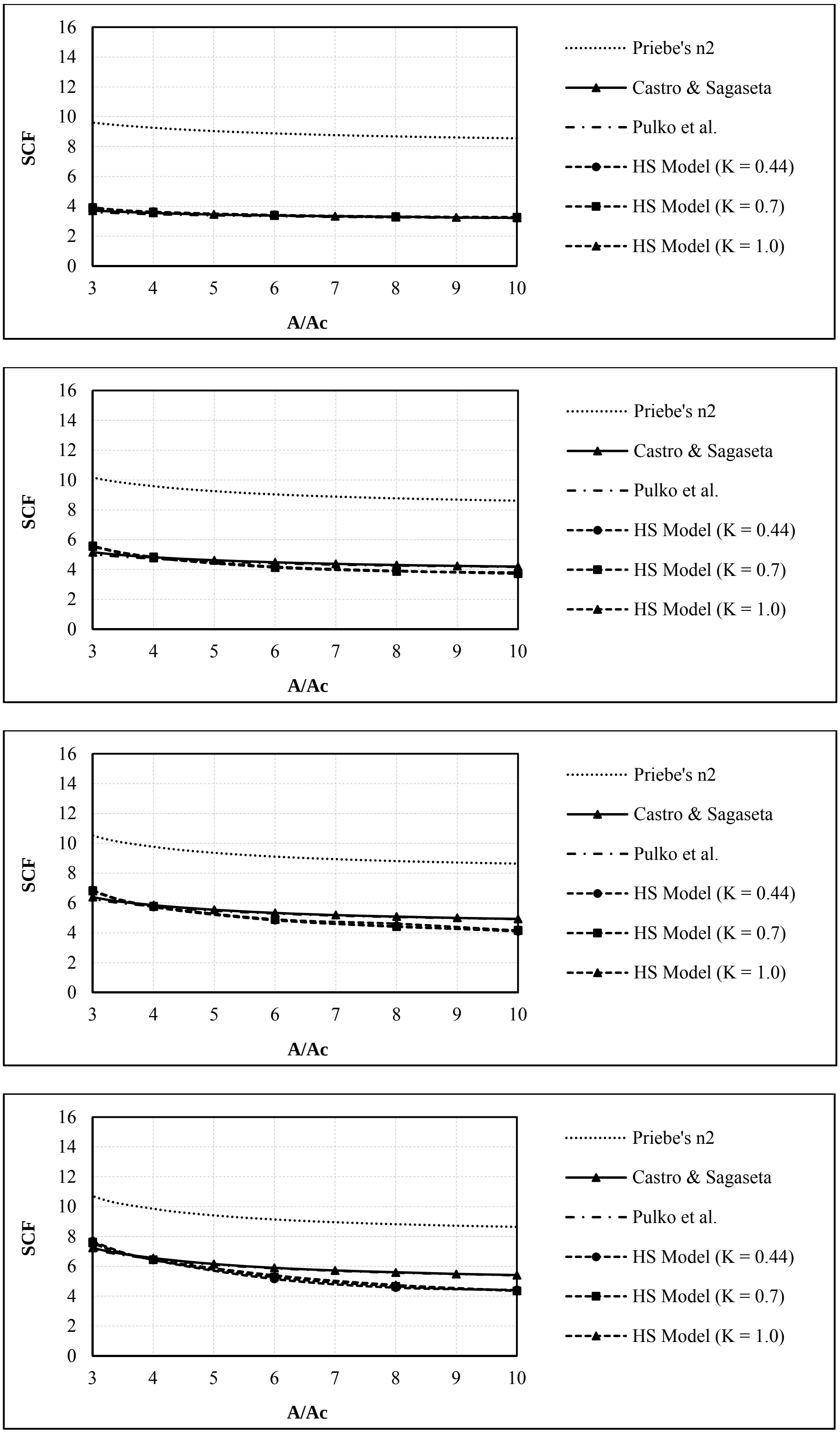


Fig. 19 SCF versus A/A_c (influence of K_0); (a) $E_c/E_s = 5$ (b) $E_c/E_s = 10$ (c) $E_c/E_s = 20$ (d) $E_c/E_s = 40$